Calculation of Spin Conduction Matrix in Spin Circuit Considering Nano-Magnetic Nodes and Copper Nano-Channel

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Abstract: In this work, we intend to calculate spin conduction matrixes of T-shaped spin circuits’ branches. Also we investigate simultaneously effects of gold nano-channel length and cross section area variations on its non-zero elements. Our findings show that $G_{11}$ and $G_{22}$ elements of series branches increase with simultaneously channel length reduction and channel cross section area grow up. For spin flip branches, $G_{22}$ element decreases with simultaneously nano-channel length and cross section area reduction. We choose copper metal as nano-channel because of its high conductance and lattice constant.

Keywords: Circuit-spin-current-conduction-nano-channel.

Introduction

Electronics of spin or spintronics is a new-fangled field which its purpose is to study the role of electron’s spin in solid-state devices. Spintronic devices require spin current [1]. Spintronics also benefits from a large class of emerging materials, such as ferromagnetic semiconductors [2,3], organic semiconductors [4], organic Ferro magnets [5,6], high temperature superconductors [7], and carbon nanotubes [8,9], which can bring novel functionalities to the traditional devices. Spin current is a difference between spin-up and spin-down electric currents. Comfortable way to create spin current is passing electron current form ferromagnetic materials. Spin injectors are materials which create spin-polarized electron injection. In this paper, we intend to calculate spin conduction matrixes of T-shaped spin circuits’ branches. For achieving this aim, we start with description of laws governing on spin circuits (section 2). Section 3 is devoted to calculation of spin conduction matrixes of T-shaped spin circuit. Also we investigate effects of gold nano-channel length and cross section area variations on its non-zero elements and Spin conduction through nano-magnetic node and non-magnetic channel interface (section 4). Finally, we present conclusion and discussion in section 5.

2. Laws governing on spin circuits:

In spin circuit flows spin current, in addition to electron current. From point of view Physics, Nano- magnetic node is defined as a collection of physical points in a device or a circuit where all the quantities of interest for spin and charge transport are at equilibrium [2]. Spin circuit obeys spin ohm’s law and spin circuit theory. Spin ohm’s law shows linear relation between spin current and spin voltage difference vector. Mathematical form of this law is [2]

$$\vec{I}_s = G \Delta \vec{V}_s$$

(1)

Where $\vec{I}_s$, $G$, $\Delta \vec{V}_s$ are spin current vector, spin conduction matrix and spin voltage vector, respectively. Spin conduction relates spin current vector to spin voltage difference vector. It can be written as [2]:

$$G = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix}$$

(2)

Spin current vector in a branch of a spin circuit can be represented as

$$\vec{I}_s = \begin{bmatrix} I_{sx} \\ I_{sy} \\ I_{sz} \end{bmatrix}$$

(3)

Where $I_{sx}$, $I_{sy}$ and $I_{sz}$ is vector spin current components along x-direction, y-direction and z-direction, respectively. Mathematical form spin voltage vector describe as

$$\vec{V}_s = \begin{bmatrix} V_{sx} \\ V_{sy} \\ V_{sz} \end{bmatrix}$$

(4)

Where $V_{sx}$, $V_{sy}$ and $V_{sz}$ spin voltage vector components along three direction, respectively. In a linear spin circuit, sum of voltage differences in any closed loop is zero. This well-known law is called Kirchhoff’s voltage law [3]. Kirchhoff’s current law says sum of the spin currents vector entering node is equal to total dissipated spin current vector from it.

3. II and T-shaped spin current

In this section, we intend to calculate spin conduction matrix of T-shaped spin circuit. As shown in Fig.1, our spin circuit consisting of two magnetic nodes, and , connected by a non-magnetic channel with cross section area , resistivity , length , spin-flip length of the channel material . This circuit has two spin-flip branches and one series branch.
In Fig. 1, we can see different parameters such as $G_{\text{se}}$ and $G_{\text{sf}}$ which are $\Pi$-shaped spin circuit spin conduction matrices of series and spin-flip branches, Also $G_{\text{seT}}$ and $(G_{\text{sfT}})$ which are T-shaped spin circuits spin conduction matrices of series and spin-flip branches, respectively. We consider $G_{\text{se}}$ and $G_{\text{sf}}$ as \[ G_{\text{se}} = \begin{bmatrix} \frac{A}{\rho} \csc \frac{L}{\lambda} & 0 & 0 \\ 0 & \frac{A}{\rho} \csc \frac{L}{\lambda} & 0 \\ 0 & 0 & \frac{A}{\rho} \csc \frac{L}{\lambda} \end{bmatrix} \] \[ G_{\text{sf}} = \begin{bmatrix} \frac{A}{\rho} \tanh \frac{L}{2\lambda} & 0 & 0 \\ 0 & \frac{A}{\rho} \tanh \frac{L}{2\lambda} & 0 \\ 0 & 0 & \frac{A}{\rho} \tanh \frac{L}{2\lambda} \end{bmatrix} \]

Where rows of matrixes demonstrate spin conductance along x, y and z direction, respectively. With transformation of $\Pi$-shaped spin circuit to those of T-shaped and because of Kirchhoff's current law, we have:

\[ G_{\text{seT}} = 2G_{\text{se}} + G_{\text{sf}} \] \[ G_{\text{sfT}} = G_{\text{sf}} G_{\text{se}}^{-1} G_{\text{sf}} + 2G_{\text{sf}} \]

Therefore spin conduction matrix of series and spin-flip branches of T-shaped spin circuit is

\[ G_{\text{seT}} = \begin{bmatrix} \frac{A}{\rho} \csc \frac{L}{\lambda} & 0 & 0 \\ 0 & \frac{A}{\rho} \csc \frac{L}{\lambda} & 0 \\ 0 & 0 & \frac{A}{\rho} \csc \frac{L}{\lambda} \end{bmatrix} \]
\[ G_{\text{sfT}} = \begin{bmatrix} \frac{A}{\rho} \csc \frac{L}{\lambda} & \frac{2A}{\rho} \csc \frac{L}{\lambda} \tanh \frac{L}{2\lambda} & 0 \\ 0 & \frac{A}{\rho} \csc \frac{L}{\lambda} & \frac{2A}{\rho} \csc \frac{L}{\lambda} \tanh \frac{L}{2\lambda} \\ 0 & 0 & \frac{A}{\rho} \csc \frac{L}{\lambda} \end{bmatrix} \]

By using experimental data for copper as nano-channel and permalloy as nano-magnetic nodes [12], we compute and investigate non-zero elements of spin conduction matrixes by taking into account simultaneously effect cross section area and length and results are shown. Figs. 2 shows $G_{11}$ element of spin conduction matrix versus simultaneous variations of cross section area and length related to series branch of (a) $\Pi$ and (b) T-shaped spin circuit.

According Figs, $G_{11}$ and $G_{22}$ element increases with simultaneously channel length reduction and channel cross section area grow up which lead to increase of current charge, spin current along x-direction and total spin current of spin circuits, respectively. Increase of $G_{33}$ and $G_{44}$ elements result in increase of spin current along y-direction and z-direction and total spin current in series branch of two spin circuits. Figs. 4 show effects of simultaneously length and cross section area on the $G_{22}$ elements of spin conduction matrixes related to spin-flip branch of spin circuits.
Fig. 4 $G_{22}$ element of spin conduction matrix versus cross section area and length variations related to spin-flip branch of (a) II and (b) T-shaped spin circuit

Figs. 4 imply that $G_{22}$ element of spin-flip branch related to two types of spin circuits, decreases with simultaneously nano-channel length and cross section area reduction which lead to decrease of spin current dissipation along x-direction and therefore loss of total spin current in series branch of two spin circuit. Also, decrease of $G_{33}$ and $G_{44}$ elements result in loss of spin current reduction along y-direction and z-direction and loss of total spin current in series branch of spin circuits. Nano-channel length reduction for a specific cross section area of channel will rise $G_{11}$ and $G_{22}$ elements related to series branch in two types of spin circuit. Of course, this process is more obvious at higher nano-channel cross section area in series branch of II-shaped spin circuit. Table 1 illustrates obtained values for all of elements of spin conduction matrices of two branches of spin circuits.

Table 1: Obtained values for all of elements of spin conduction matrices of two branches spin circuits.

<table>
<thead>
<tr>
<th>Variants</th>
<th>II-shaped</th>
<th>T-shaped</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Series branch</td>
<td>spin-flip branch</td>
</tr>
<tr>
<td>L (nm)</td>
<td>A (nm)²</td>
<td>$G_{11}$</td>
</tr>
<tr>
<td>20</td>
<td>100</td>
<td>0.7</td>
</tr>
<tr>
<td>30</td>
<td>400</td>
<td>1.9</td>
</tr>
<tr>
<td>50</td>
<td>500</td>
<td>1.4</td>
</tr>
</tbody>
</table>

4. Spin conduction through nano-magnetic node and non-magnetic channel interface

Electrical spin injection (ESI) is spin-polarized carrier injection by using a magnetic contact. In spin circuits, ESI occurs through nano-magnetic nodes which act as spin injectors. When ESI is performed, spin voltage exist which happen due to creation of spin polarized population in the node. If we look at this situation from a microscopic point of view, conduction occurs through spin-dependent reflection and transmission at interface. In this section, we want to calculate spin conduction matrix of nano-magnetic node and non-magnetic channel interface in the fixed coordinate system tied to the nano-magnetic node shape (Fig. 5).

For achieving this purpose, first we write spin ohm’s law for such system relate to the case which nano-magnetic node have magnetic moment along interface. We assume that interface be along x-direction of a right-handed coordinating system. Then we can extract spin matrix form spin ohm’s law we consider spin voltage in nano-magnetic node and non-magnetic channel as below forms

$$\overline{V}_{\text{Node}} = \begin{bmatrix} V_{x} \\ V_{y} \\ V_{z} \end{bmatrix}$$

Where $V_{x}, V_{y}$ and $V_{z}$ is non-magnetic node charge voltage, non-magnetic channel charge voltage, non-magnetic channel vector spin voltage components along x-axis, y-axis and z-axis, respectively. Since we neglect spin accumulation in nano-magnetic node, vector spin voltage components of nano-magnetic node are zero. Now, we introduce spin current from nano-magnetic node to non-magnetic channel as

$$\overline{I}_{s} = \begin{bmatrix} I_{x} \\ I_{y} \\ I_{z} \end{bmatrix}$$

Where $I_{x}, I_{y}$ and $I_{z}$ is charge current from node to channel, vector spin current components along x-axis, y-axis and z-axis from node to channel, respectively. By using spin ohm’s law, we can write mathematical form of these spin currents in terms of parallel ($I_{p}$) and perpendicular ($I_{L}$) component to magnetic moment as [8]

$$I_{c} = G_{c}(V_{y} - V_{p}) + G_{c}\hbar\overline{V}_{\text{Channel}}$$

$$I_{sl} = G_{sl} \left[ \hbar \times \left( \overline{V}_{\text{Channel}} \times \hbar \right) \right] + G_{fl} \left( \overline{V}_{\text{Channel}} \times \hbar \right)$$

Where $G_{c}, G_{sl}$ and $G_{fl}$ are sum of spin-dependent conductance of node-channel interface, spin-dependent conductance difference of node-channel interface, spin transfer conductance of channel and field-like conductance term, respectively. Terms $\hbar \overline{V}_{\text{channel}}, \overline{V}_{\text{channel}} \times \hbar$ and $\hbar \times \left[ \overline{V}_{\text{channel}} \times \hbar \right]$ demonstrate projection of the up and down components of the spin accumulation on the magnetic moment vector of node, perpendicular component of spin current in plane with magnetic moment and spin accumulation in channel and perpendicular component of spin current perpendicular to magnetic moment and spin accumulation in
channel. Then, spin conduction matrix for node-channel interface \([10, 11]\) is as

\[
G_{\text{int}} = \begin{bmatrix}
\sigma & aG & 0 & 0 \\
aG & \sigma & 0 & 0 \\
0 & 0 & G_{\text{SL}} & G_{\text{FL}} \\
0 & 0 & -G_{\text{FL}} & G_{\text{SL}} \\
\end{bmatrix}
\]  

(15)

Now, we consider a special case in which magnetic moment along of x-direction node-channel interface. Then spin conduction matrix convert to below form

\[
G_{\text{int}}(\vec{x}) = \begin{bmatrix}
G_{11} & \alpha G_{11} & 0 & 0 \\
\alpha G_{11} & G_{11} & 0 & 0 \\
0 & 0 & G_{\text{SL}} & G_{\text{FL}} \\
0 & 0 & -G_{\text{FL}} & G_{\text{SL}} \\
\end{bmatrix}
\]  

(16)

Where \(\alpha\) and \(\alpha_1\) are sum of spin-dependent conductance of the node-channel interface along x-direction, spin-dependent conductance difference of the node-channel interface along x-direction, respectively. We assume that magnetization of node is along x-direction of right-handed coordinate system. Since total spin current is independent of the orientation of the interface, thus we can use equation (16) for any direction of magnetic moment as long as \(\vec{m} = \vec{x}\). Now, we want to derive the spin conduction matrix for a free layer nano-magnetic node with an arbitrary magnetic moment direction. We select \(\varphi\) as an angle between magnetic moment direction and z-direction and \(\theta\) as an angle between xy-plane projection of magnetic moment direction on xy-plane and x-direction. Magnetic moment in coordinate system xyz is as

\[
\vec{m} = \cos \theta \sin \varphi \hat{x} + \sin \theta \sin \varphi \hat{y} + \cos \varphi \hat{z}
\]

We choose a new coordinate system \(x'y'z'\) in which magnetic moment direction is collinear with the new x-direction

\[
\begin{align*}
\hat{x}' &= \cos \theta \sin \varphi \hat{x} + \sin \theta \sin \varphi \hat{y} + \cos \varphi \hat{z} \\
\hat{y}' &= -\cos \varphi \cos \theta \sin \varphi \hat{y} + \sin \varphi \cos \theta \sin \varphi \hat{x} \\
\hat{z}' &= \cos \theta \cos \varphi \hat{z} - \sin \theta \cos \varphi \hat{x}
\end{align*}
\]

Therefore, spin conduction matrix for node-channel interface with an arbitrary magnetic moment direction of node is written as

\[
G_{\text{int}}(\vec{m}) = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\
\sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} \\
\sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} \\
\end{bmatrix}
\]  

(17)

Where

\[
\begin{align*}
\sigma_{11} &= G_{11} \\
\sigma_{12} &= aG_{11} \cos \theta \sin \varphi \\
\sigma_{13} &= aG_{11} \sin \theta \sin \varphi \\
\sigma_{14} &= aG_{11} \cos \varphi \\
\sigma_{21} &= \cos \varphi \sin \theta \alpha G_{11} \\
\sigma_{22} &= \sin \varphi [\cos \theta \sigma_{11} + \sin \theta G_{\text{SL}}] + \cos \theta \sigma_{11} \\
\sigma_{23} &= \sin \theta \sin \varphi \sigma_{11} - \sigma_{13} G_{\text{FL}} + \cos \varphi G_{\text{FL}} \\
\sigma_{24} &= \cos \theta \sin \varphi \sigma_{11} - \sigma_{14} G_{\text{FL}} \\
\sigma_{31} &= \sin \theta \sin \varphi \alpha G_{11} \\
\sigma_{32} &= \cos \theta \sin ^2 \varphi \sin \theta \sigma_{11} - \cos \varphi G_{\text{FL}} \\
\sigma_{33} &= \sin \theta \sin ^2 \varphi G_{\text{SL}} + \cos \theta \sin ^2 \varphi G_{\text{FL}} + \left[\cos \varphi + \sin \theta \sin \varphi\right] G_{\text{FL}} \\
\sigma_{34} &= \sin \theta \sin ^2 \varphi G_{\text{SL}} + \cos \theta \sin ^2 \varphi G_{\text{FL}} \\
\sigma_{41} &= a \cos \theta \sigma_{11} \\
\sigma_{42} &= \cos \theta \sin \varphi \sigma_{11} + \sin \theta \sin \varphi G_{\text{FL}} \\
\sigma_{43} &= \sin \theta \sin \varphi \sigma_{11} - \cos \theta \sin \varphi G_{\text{FL}} \\
\sigma_{44} &= \cos ^2 \varphi \sigma_{11} + [\sin \theta \sin \varphi \cos \varphi + \cos \theta \sin ^2 \varphi \cos ^2 \varphi] G_{\text{FL}} \\
\end{align*}
\]

5. Conclusion and Discussion

According to our results for two spin circuit’s series branches, we find out that \(G_{11}\) and \(G_{22}\) elements increase with simultaneously channel length reduction and channel cross section area grow up which lead to increase of current charge, spin current along x-direction and total spin current, respectively. While \(G_{22}\) element related to spin-flip branch of two types of spin circuits decreases with simultaneously nano-channel length and cross section area reduction which lead to decrease of spin current dissipation along x-direction and therefore loss of total spin current in series branch of two spin circuit. Calculation of spin conduction matrix is important for total spin current vector components calculation. Our goal of write spin conduction matrix of node-channel interface was calculation of total spin current vector components. By parametric
computing of these components, we can interpreted, how we can increase total spin current vector and as its result increase of electrically spin injection and spin polarization which can imply better application of spintronics devices.

References